# Technical note 1 <br> The concept of impedance. Input impedance of a sEMG amplifier and its relevance in sEMG detection. 

Roberto Merletti<br>LISiN, Dept. of Electronics and Telecommunications, Politecnico di Torino, Italy roberto.merletti@formerfaculty.polito.it or roberto@ robertomerletti.it www.robertomerletti.it

This technical note is aimed to readers with a high-school knowledge of mathematics and physics of electrical phenomena. For this reason, some concepts are oversimplified and may not be rigorously correct.

## 1. The concept of voltage generator

A voltage generator is a device that has two terminals (e.g. the two poles of a battery or the two poles of the socket of the power line) that accumulates electric charges on one pole, removing them from the other. This voltage is the analog of pressure in a hydraulic circuit and is measured in Volt (V). It can be constant or variable in time. A particular case of constant ("Direct Current" or DC) voltage generator is a battery. A particular case of time-variable voltage is the sinusoidal voltage present between the poles of a power line socket in our homes. Another particular case is the time variable voltage of a bioelectric signal (e.g. ECG, EMG, EEG).

In mathematical terms this can be expressed as:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{o}}=\mathrm{k} \tag{1}
\end{equation*}
$$

where k is a constant, e.g. 1.5 V for a 1.5 V battery,

$$
\begin{equation*}
\mathrm{Vo}_{\mathrm{o}}(\mathrm{t})=\mathrm{A} \cdot \sin (2 \pi \mathrm{ft})=\mathrm{A} \cdot \sin (2 \pi \mathrm{t} / \mathrm{T}) \tag{2}
\end{equation*}
$$

where $\mathrm{vo}_{\mathrm{o}}$ is the generator voltage, $\sin (2 \pi \mathrm{ft})$ is a sinusoid having frequency f (measured in cycles/s or Hz ), and period of $\mathrm{T}=1 / \mathrm{f}$ seconds, A is the peak value of the sinusoid in Volt, $\pi=3.14 \ldots$, and t is time in seconds. A special case is the power line voltage whose frequency is 50 Hz ( $\mathrm{T}=20 \mathrm{~ms}$ ) in Europe and $60 \mathrm{~Hz}(\mathrm{~T}=16.67 \mathrm{~ms}$ ) in other countries.

$$
\begin{equation*}
\mathrm{V}_{\mathrm{o}}(\mathrm{t})=\mathrm{f}(\mathrm{t}) \tag{3}
\end{equation*}
$$

where the symbol $f(t)$ means "a function of time", as in the case of ECG, EMG, EEG.
Fig. 1a, b, c show an example for each of the three cases. The expression $v_{o}(t)=f(t)$ is called $a$ "signal". A signal, whose duration is T seconds, can be mathematically expressed as a sum of N sinusoids having periods $T, T / 2, T / 3, T / 4 \ldots T / N$ and frequencies $f_{1}=1 / T, f_{2}=2 / T, f_{3}=3 / T$, $f_{4}=4 / T \ldots f_{N}=N / T$. These "sinusoidal components" are purely mathematical tools, are called "harmonics" or "spectral lines", or "frequency components". They have no direct physiological meaning, and represent the mathematical "Fourier series expansions" or "Fourier series" of the signal. See and https://www.robertomerletti.it/en/emg/material/teaching/module4 and Technical note 2 on "Fourier series expansion of a signal". Fig. 1d shows the symbols used to depict a battery and a generic voltage generator whose voltage is a function of time.

If a "load" is connected to a voltage generator, for example a light bulb to a battery, a heater to the power line, an amplifier to a pair of EMG, ECG or EEG electrodes, a closed circuit will be created and electrical charges will flow in it, conventionally from the positive (+) to the negative (-) pole of the generator. This flow of charges is the "current intensity", or simply the "current", circulating in the circuit. Considering a hydraulic analog model, the pressure generated by a pump is the dual of the voltage produced by the electric generator and the flow of fluid in the piping is the dual of the electric current (see https://www.robertomerletti.it/en/emg/material/teaching/module3).

## 2. The concepts of impedance and resistance. Ohm's Law.

The amount of current flowing in the load connected to the generator depends on the "obstacle" encountered by the flow of electric charges in their movement. Current is the flow of charges per unit time and is measured in Coulomb/s or Ampere. This obstacle is called "impedance", indicated with the symbol Z and measured in $\operatorname{Ohm}(\Omega)$. The larger is the impedance the smaller is the flow. There are three types of circuit elements (loads) presenting three types of impedance: resistors, capacitors and inductors. Only the first two are considered here.

Resistive impedance. Resistive impedance (or simply "resistance") is provided by resistors. A "resistor" (usually improperly called a "resistance") is a device offering a "friction-type impedance" that is independent of frequency, like the impedance offered by a narrow pipe to the flow of a fluid. The impedance of a resistor is its resistance measured in Ohms $(\Omega)$. Its graphic symbol in indicated in Fig 1e,f.

Capacitive impedance. Consider blowing air in a stiff or a compliant rubber balloon: the stiff balloon offers a larger impedance than the compliant one. This is not a "friction-type" impedance, it is a "stiffness-type" impedance. This impedance depends on the frequency $f$ of the voltage provided by the generator and on the nature of the "obstacle". The electrical device presenting this type of impedance is a capacitor. A "capacitor" (often improperly called a "capacitance") is a physical device made of two surfaces (called "armatures") facing each other and separated by and insulating layer that cannot be "crossed" by electric current. Its symbol is indicated in Fig 1g.

The two armatures can be two parallel plates (as indicated in most textbooks) or the electrical wires in the wall and the surface of a person in a room. An electric capacitor is analog to an elastic balloon.
A large capacitor is analog to a compliant balloon and a small capacitor is analog to a stiff balloon. A large capacitors has large facing surfaces close to each other and has low impedance. A small capacitor has small facing surface at some distance from each other and has high impedance. The facing surfaces can be imagined as charge containers being filled and emptied, just as two balloons are fluid containers being filled and emptied as described in slides 7, 8 and 20 of Teaching Module 3, https://www.robertomerletti.it/it/emg/material/teaching/module3. The current does not flow through the insulating material separating the plates but back and forth from one to the other (the issue of "displacement current" is not addressed here). In case of a sinusoidal voltage generator (eq. 2 and Fig. 1b) the current will have alternate directions (AC current) and the obstacle will depend on the nature of the circuit (for example a balloon will behave differently from a pipe) and on the frequency of the sinusoidal voltage. Capacitors are measured in Farads ( F ) and their impedance is measured in Ohms ( $\Omega$ ). A Farad is a large unit and picoF (10-12 F), nanoF (10-9 F), and microF $(10-6 \mathrm{~F})$ are more common submultiples.

The relationship described in eq. 4 and eq. 5 is known as Ohm's Law.

$$
\begin{equation*}
\mathrm{i}=\mathrm{V}_{\mathrm{o}} / \mathrm{R} \tag{4}
\end{equation*}
$$

(valid if the impedance is purely resistive. In this case i and vo have the same waveshape)

$$
\begin{equation*}
\mathrm{i}=\mathrm{V}_{\mathrm{o}} / \mathrm{Z} \tag{5}
\end{equation*}
$$

(valid in general; Z is a function of frequency and is different for each harmonic of vo , therefore i and vo do not have the same waveshape because the spectrum of $i$ is not the same as the spectrum of $\mathrm{V}_{\mathrm{o}}$.


Fig. 1. a) constant (or DC) voltage, such as that produced by a battery, b) sinusoidal voltage, such as that of the power line, c) general waveform, such as that of sEMG, d) symbols of a battery and of a generic generator, e) battery and generic generator connected to a resistors, $f$ ) and $g$ ) symbols used to indicate a resistor and a capacitor.

## 3. Resistors and capacitors.

A resistor is a "friction-type" element, as described above. A capacitor has the more complex behavior described below (see also https://www.robertomerletti.it/en/emg/material/teaching/module3).

The quantity of fluid contained in an elastic balloon is equal to the pressure inflating the balloon multiplied by its compliance. Similarly, the charge q accumulated on the armature of a capacitor having capacitance C is equal to the voltage v between the armatures multiplied by the capacitance, that is

$$
\begin{equation*}
\mathrm{q}(\mathrm{t})=\mathrm{Cv}(\mathrm{t}) \tag{6}
\end{equation*}
$$

The unit of measurement of C is therefore Coulomb/Volt, which is called Farad (see above). The current flowing in and out of $C$ is the rate of change of $q$, that is the derivative of $q$ with respect to time. Taking the derivative of both sides of eq. 6 we have:

$$
\begin{equation*}
i(t)=\frac{d q(t)}{d t}=C \frac{d v(t)}{d t} \tag{7}
\end{equation*}
$$

Eq 7 shows that if v is constant, that is $\frac{d v(t)}{d t}=0$, then $\mathrm{i}(\mathrm{t})=0$. No current flows in a capacitor if the voltage applied to it is constant and the capacitor is "charged". No fluid flows in a balloon if the pressure applied to it is constant and the balloon is full because the back pressure equals the applied pressure.

If the voltage applied to the capacitor is a sinusoid $\mathrm{v}(\mathrm{t})=\mathrm{A} \cdot \sin (2 \pi \mathrm{ft})$ the current will be proportional to the derivative of $\sin (2 \pi \mathrm{ft})$ (which is $2 \pi \mathrm{f} \cos (2 \pi \mathrm{ft})$ ) so that $\mathrm{i}(\mathrm{t})=\mathrm{A} \cdot \mathrm{C} \cdot 2 \pi \mathrm{f} \cdot \cos (2 \pi \mathrm{ft})$ (from eq.7). If we consider the ratio of the amplitudes (indicated with capital letters) V/I we have

$$
\begin{equation*}
\mathrm{V} / \mathrm{I}=\mathrm{A} /(2 \pi \mathrm{f} \mathrm{AC})=1 /(2 \pi \mathrm{fC})=\mathrm{Z}_{\mathrm{c}} \tag{8}
\end{equation*}
$$

Eq 8 defines the impedance of a capacitor. Note that this impedance is inversely proportional to the frequency $f$ of the sinusoidal voltage and to the capacitance $C$ of the capacitor. For example, the impedance of a 5 pF capacitor at the frequency of 50 Hz will be

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{c}}=1 / 2 \pi \mathrm{fC}=1 /(2 \cdot 3.14 \cdot 50 \cdot 5 \cdot 10-12)=637 \cdot 10_{6}=637 \mathrm{M} \Omega(\text { for } \mathrm{f}=50 \mathrm{~Hz}) \tag{9}
\end{equation*}
$$

While, at the frequency of 60 Hz , the impedance of a 5 pF capacitor will be

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{c}}=1 / 2 \pi \mathrm{fC}=1 /(2 \cdot 3.14 \cdot 60 \cdot 5 \cdot 10-12)=530 \cdot 10_{6}=530 \mathrm{M} \Omega(\text { for } \mathrm{f}=60 \mathrm{~Hz}) \tag{10}
\end{equation*}
$$

If the capacitance doubles, the impedance becomes half. If the frequency doubles, the impedance becomes half.

The above considerations apply strictly to the case of sinusoidal voltages. In case of a voltage generator producing a generic, non-sinusoidal, voltage function of time, that is the signal $v_{o}(t)=f(t)$ (eq. 3), the concept of impedance is applicable to each of the harmonics (or spectral lines) of the signal mentioned in the Introduction. The concept is not applicable to the entire signal and it is meaningless and incorrect to talk about a numerical impedance value offered by a load to the ECG or EMG or EEG signal generators since such impedance is different for the different sinusoidal harmonics composing the signal. Equations 4 and 5 define the relation between voltage, current and impedance and are expressing the concept that the current flowing in a circuit is directly proportional to the voltage causing it, and inversely proportional to the impedance offered by the circuit (at each frequency).

Nevertheless, the expression "impedance value offered by a load to the ECG or EMG or EEG signal" is frequently used and should be read as "impedance value offered by a load to each of the
harmonics of the ECG or EMG or EEG signal". If the load is capacitive, this impedance is different for each harmonic because it is inversely proportional to the frequency of the harmonic since $\mathrm{Z}_{\mathrm{c}}=1$ / $2 \pi \mathrm{f} \mathrm{C}$ (eq. 8 and Fig. 2a). Therefore, the impedance is low for high frequency harmonics and high for low low frequency harmonics of the signal. This is the reason why an impedance numerical value should always be associated to a specific frequency.

Fig. 2a shows a generator with a purely capacitive load, Fig.2b shows a generator with a load made by a capacitor and a resistor in parallel (a model of the input impedance of a bioelectric signal amplifier) and Fig.2c shows the currents flowing in R and C and their sum in the case of a sinusoidal generator. As explained below eq. 7, the two currents ic and ir are time shifted by a quarter of a cycle. For this reason, the peak value of $i$ is not the sum of the peak values of ir and ic. Depending on the value of C and on the frequency, $\mathrm{Z}_{\mathrm{c}}$ may be smaller, equal or greater than R .


Fig. 2. Capacitive loads. a) circuit including a generic generator and a capacitive load, b) circuit with a load including a resistor and a capacitor in parallel: $\left.i=i_{R}+i c,, c\right)$ plot of the sinusoidal currents $i$, $i_{R}$, and $i c$, when $v_{A B}$ is a sinusoidal voltage, that is $f(t)=v_{A B}(t)=v_{o}(\mathrm{t})=\mathrm{A} \cdot \sin (2 \pi \mathrm{ft})$. Note that the peak value of $i$ is not the sum of the peak values of $i$ in and $i c$. Also note that $R$ is not affected by frequency but $Z_{c}$ is.

### 3.1. Resistor-capacitor combinations.

A resistor and a capacitor can be in series or in parallel. The parallel configuration is the one of interest in our case.

If a sinusoidal voltage $v A B=A \sin (2 \pi f t)$ is applied to the terminals $A$ and $B$ of the load, as in Fig. $2 \mathrm{~b}, \mathrm{a}$ current $\mathrm{iR}=\mathrm{A} \cdot \sin (2 \pi \mathrm{ft}) / \mathrm{R}$ will flow through the resistor and a current ic $=\mathrm{A} \cdot \cos (2 \pi \mathrm{ft}) / \mathrm{Zc}$ will flow through the capacitor and their sum will be $\mathrm{i}=\mathrm{A} \cdot \sin (2 \pi \mathrm{ft}) / \mathrm{R}+\mathrm{A} \cdot \cos (2 \pi \mathrm{ft}) / \mathrm{Zc}$.

Their peak values will be $\mathrm{i}_{\mathrm{R}}=\mathrm{A} / \mathrm{R}$ and $\mathrm{ic}=\mathrm{A} / \mathrm{Zc}$, respectively, but the peak value of their sum i will not be $A / R+A / Z c$ because the two sinusoids are out of phase by $90_{0}$ (a quarter of a period, since one is a sine and the other is a cosine) as indicated in Fig. 2c. The impedance Z of the RC group in Fig 2 b is given by $Z_{i}(f)=\frac{R_{i}}{\sqrt{1+\left(2 \pi f R_{i} C_{i}\right)^{2}}}$ and is a function of the frequency f .
If the voltage $v_{A B}$ is not a sinusoid, the above equation applies to each of its sinusoidal harmonics.

### 3.2. Voltage divider

Fig. 3a shows the concept of a voltage divider. According to eq. 4, the voltage vin is applied to two resistor in series causing the flow of the current $i=\operatorname{Vin} /\left(R_{1}+R_{2}\right)$. Therefore, the voltage on $R 2$ is:

$$
\begin{equation*}
\text { Vout }=\mathrm{i} \cdot \mathrm{R}_{2}=\operatorname{Vin} \cdot \mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right) \tag{11}
\end{equation*}
$$

which is the voltage divider equation. This equation means that a fraction of vin is present at the output of the device depicted in Fig. 3a and this fraction is given by $R_{2} /\left(R_{1}+R_{2}\right)$. For example: let us consider the cases R1 $=$ R2; application of eq. 11 indicates that vout $=0.5 \mathrm{vin}$, that is $50 \%$ of Vin.

Let us further consider the case $\mathrm{R}_{1}=10 \mathrm{k} \Omega$ and $\mathrm{R}_{2}=990 \mathrm{k} \Omega$; this implies that vout is $99 \%$ of vin. Therefore, if $\mathrm{R}_{1} \ll \mathrm{R}_{2}$, vout will be almost equal to vin. With some complications that will be neglected here, the same considerations apply if, instead of $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ we have two impedances, $\mathrm{Z}_{1}$ and $\mathrm{Z}_{2}$. In case of sinusoidal vin we have to account for the fact that vout will not, in general, be in phase with vin.

In case of a general signal $\operatorname{Vin}(t)=f(t)$ the same considerations apply to each harmonic of $f(t)$ and the fraction of $f(t)$ present at the output will be different for different frequencies. The foltage divider will therefore perform as a filter.


Fig. 3. Voltage divider. a) concept of voltage divider made of two resistors, b) double voltage divider with impedances, c) input impedance of a bioelectric signal amplifier, d) circuit for the analysis of the common mode voltage present at the input of a bioelectric signal amplifier.

### 3.3. Double voltage divider

Let us now consider two voltage dividers with the same vin $=$ vcm, as indicated in Fig. 3b.
One is formed by $Z_{e 1}$ and $Z_{i}$ and the other is formed by $Z_{e 2}$ and $Z_{i}$. The reason for these subscripts is that, in the case of a bioelectric signal amplifier, $\mathrm{Z}_{\mathrm{e} 1}$ and $\mathrm{Z}_{\mathrm{e} 2}$ are the contact impedances of two electrodes and $\mathrm{Z}_{\mathrm{i}}$ is the input impedance of the amplifier. This will further be discussed in section 4.

The two voltage dividers provide outputs vout1 and vout respectively. These two voltages are not the same because, in general, $\mathrm{Z}_{\mathrm{el}}$ and $\mathrm{Z}_{\mathrm{e} 2}$ are different. This results in a differential voltage $\mathrm{v}_{\mathrm{diff}}$ between the two outputs Vout2 and Vout1 (Fig. 3b):
$V_{\text {diff }}=V_{\text {out } 2}-V_{\text {out }}=v_{c m} \cdot Z_{i} /\left(Z_{\mathrm{e} 2}+Z_{i}\right)-V_{c m} \cdot Z_{i} /\left(Z_{\mathrm{e} 1}+Z_{i}\right) \cong v_{c m} \cdot\left(Z_{\mathrm{e} 2}-Z_{\mathrm{e} 1}\right) / Z_{i}$
where $\cong$ means "about equal" and vcm is the "common mode" input voltage to the two dividers.
For example, consider the resistive impedances $Z_{\mathrm{e} 2}=\mathrm{R}_{\mathrm{e} 2}=200 \mathrm{k} \Omega$, $\mathrm{Z}_{\mathrm{e} 1}=\mathrm{R}_{\mathrm{e} 1}=100 \mathrm{k} \Omega$,
$\mathrm{Z}_{\mathrm{i}}=\mathrm{R}_{\mathrm{i}}=100 \mathrm{M} \Omega$, and $\mathrm{vcm}=1 \mathrm{~V}$.
Application of eq. 12 will give $V_{\text {diff }}=1 \cdot 100 \cdot 10_{3} / 100 \cdot 10_{6}=10-3 \mathrm{~V}=1 \mathrm{mV}$. It is very important to understand that this differential voltage is caused by the common mode voltage and by the fact that
the values of $\mathrm{Z}_{\mathrm{e} 2}$ and $\mathrm{Z}_{\mathrm{e} 1}$ are not identical. The two input impedances $\mathrm{Z}_{\mathrm{i}}$ of the amplifier may considered identical.

## 4. The "front-end" stage of a bioelectric signal amplifier: effect of input impedance

A differential bioelectric signal amplifier (e.g. a sEMG or ECG or EEG amplifier) has some important characteristics that affect its performance and therefore the properties of the output signal. Such characteristics are:
The differential amplification (or gain) Ad: this is the factor by which the differential input signal is multiplied to make it bigger and compatible with the accepted input voltage range of the $\mathrm{A} / \mathrm{D}$ converter.

The common mode amplification (or gain) Acm: this is the factor by which the common mode input signal (that is the signal common to the two inputs) is unfortunately multiplied. The ideal value for a perfect amplifier would be $\mathrm{A}_{\mathrm{cm}}=0$ but the real values are in the range 0.01-0.1. That means that the common mode input signal is attenuated but is not eliminated.

The common mode rejection ratio (CMRR): this is the ratio $\mathrm{Ad} / \mathrm{Acm}$ and is usually expressed as $20 \log _{10}\left(\mathrm{Ad} / \mathrm{Acm}_{\mathrm{cm}}\right)$ and measured in decibel $(\mathrm{dB})$. For example if $\mathrm{Ad} / \mathrm{Acm}_{\mathrm{cm}}=1000 / 0.1=10000=104$, then $\mathrm{CMRR}=80 \mathrm{~dB}$.

The input impedance: this is an important characteristic because it affects the degree of power line interference. If we "look" into each of the two inputs of the amplifier, we "see" a large resistor in parallel with a small capacitor, as indicated in Fig. 2b and 3c. The ratio between the applied voltage and the current flowing into the amplifier input is $\mathrm{v} / \mathrm{i}=\mathrm{Zi}$. Since the amplifier has two inputs, there are two input impedances that are very similar and can be considered identical. Let us consider their effect on the signal.

### 4.1 The input impedance of a sEMG amplifier: its effect on the sEMG signal

A differential bioelectric signal amplifier is usually powered by two DC voltages $\mathrm{V}_{\mathrm{b}+}$ and $\mathrm{V}_{\mathrm{b}}$ whose center point $(0 \mathrm{~V})$ is the reference for all local voltages. It also has two inputs and two identical input impedances, one for each input, that are indicated with $\mathrm{Zi}_{\mathrm{i}}$. They are internal, within the amplifier, but in Fig. 3d they are represented outside of the amplifier for graphical convenience. When a voltage $v$ is applied to one of these inputs a current $\mathrm{i}=\mathrm{v} / \mathrm{Zi}$ will flow into the amplifier. The input impedance consists of a large resistor (order of 1-10 G $\Omega$, that is $10_{9}-10_{10} \Omega$ ) in parallel with a small capacitor (order of 2-10 pF). Each electrode-skin impedance $\mathrm{Ze}_{\mathrm{e}}$ and the amplifier's input
impedance $\mathrm{Z}_{\mathrm{i}}$ form a voltage divider whose total impedance is very high so that the current flowing through them is of the order of a fraction of pA . For frequencies in the bandwidth of the sEMG ( $20-400 \mathrm{~Hz}$ ) the impedance offered by the input capacitor will be considerably lower than that offered by the input resistor and the input current will flow mostly in it (that is ic $\gg \mathrm{i}$ ). In other words, the input impedance of the amplifier, for frequencies in the sEMG range $(20-400 \mathrm{~Hz})$ will be primarily determined by the input capacitor, not by the input resistor. Table 1 provides numerical examples.

Table 1. Values of the impedance in Ohms ( $\Omega$ ) offered by the input capacitor of an amplifier for different values of frequency and capacitance. The values are computed as $\mathrm{Z}_{\mathrm{c}}=1 / 2 \pi \mathrm{f} \mathrm{C}$ (eq. 8). $(1 \mathrm{pF}=1 \cdot 10-12 \mathrm{~F}, \quad 1 \mathrm{G} \Omega=1 \cdot 109 \Omega)$

| $\mathrm{f} \downarrow \mathrm{C} \rightarrow$ | $\mathrm{C}=2 \mathrm{pF}$ | $\mathrm{C}=5 \mathrm{pF}$ | $\mathrm{C}=10 \mathrm{pF}$ |
| :---: | :---: | :---: | :---: |
| 20 Hz | $\mathrm{Z}_{\mathrm{c}}=3.98 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=1.59 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.80 \mathrm{G} \Omega$ |
| 50 Hz | $\mathrm{Z}_{\mathrm{c}}=1.60 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.64 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.32 \mathrm{G} \Omega$ |
| 100 Hz | $\mathrm{Z}_{\mathrm{c}}=0.80 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.32 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.16 \mathrm{G} \Omega$ |
| 200 Hz | $\mathrm{Z}_{\mathrm{c}}=0.40 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.16 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.08 \mathrm{G} \Omega$ |
| 300 Hz | $\mathrm{Z}_{\mathrm{c}}=0.26 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.10 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.05 \mathrm{G} \Omega$ |
| 400 Hz | $\mathrm{Z}_{\mathrm{c}}=0.20 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.08 \mathrm{G} \Omega$ | $\mathrm{Z}_{\mathrm{c}}=0.04 \mathrm{G} \Omega$ |

The input capacitor impedance values of Table 1 are in parallel to a resistance value of 1-10 G $\Omega$ (Fig. 2b and Fig. 3c). This parallel path reduces the total input impedance. In any case, the input impedance of the amplifier is higher than $0.04 \mathrm{G} \Omega$ (worst case is $40 \mathrm{M} \Omega$ for $\mathrm{C}=10 \mathrm{pF}$ and $\mathrm{f}=400$ Hz outlined in yellow) for the entire sEMG bandwidth ( $20-400 \mathrm{~Hz}$ ). For input capacitors $\mathrm{C} \geq 5 \mathrm{pF}$ and frequencies
$\mathrm{f} \geq 50 \mathrm{~Hz}$ the input impedance $\mathrm{Z}_{\mathrm{i}}$ is determined by the impedance $\mathrm{Z}_{\mathrm{c}}$ of input capacitor and NOT by the input resistance R because $\mathrm{Z}_{\mathrm{c}} \ll \mathrm{R}$ and most of the input current flows in C and not in R (Fig. 3 c ).
For greater accuracy, the impedance of the parallel combination of $\mathrm{R}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{i}}(\mathrm{Fig} .3 \mathrm{c})$ is given by the equation:

$$
\begin{equation*}
Z_{i}(f)=\frac{R_{i}}{\sqrt{1+\left(2 \pi f R_{i} C_{i}\right)^{2}}} \tag{13}
\end{equation*}
$$

where the term $\left(2 \pi f R_{i} C_{i}\right)^{2}$ is usually much greater than 1 , resulting in $Z_{i} \cong Z_{c}=1 / 2 \pi f C_{i}$.
When the skin is properly treated, in the sEMG frequency range (that is for all the harmonics of the sEMG signal from 20 Hz to 400 Hz ), the electrode-skin interface offers an impedance Ze in the range of $10 \mathrm{k} \Omega$ to $100 \mathrm{k} \Omega$, much lower than the input impedance of the amplifier. Therefore, in the worst case (for $\mathrm{f}=400 \mathrm{~Hz}$ ), the voltage divider $\mathrm{Z}_{\mathrm{e} 2}$ and $\mathrm{Z}_{\mathrm{i}}$ and the voltage divider $\mathrm{Z}_{\mathrm{e} 1}$ and Zi (Fig. 3b and

3d), attenuate the sEMG signal by a factor of about 0.99 ( $1 \%$ amplitude reduction) which is not an issue.

We can then conclude that:

1. In the frequency band of the sEMG signal, the input impedance of a bioelectric amplifier is largely determined by the input capacitance and NOT by the input resistance. The value of the input resistance R is not relevant as long as $\mathrm{R} \gg 1 \mathrm{G} \Omega$, as is the case for any good bioelectric amplifier.
2. Manufacturers should report the value of the input capacitor together with that of the input resistor. Reporting only the input resistance does not provide information about Zi .
3. The sEMG signal is attenuated by the voltage divider Ze and Zi only if $\mathrm{Ze}_{\mathrm{e}}$ is of the order of many $\mathrm{M} \Omega$, that is the electrode-skin contact is extremely poor. In general, for a reasonably good electrode-skin contact, this attenuation is negligible (less than $1 \%$ ) because $\mathrm{Ze}_{\mathrm{e}} \ll \mathrm{Zi}_{\mathrm{i}}$.

However, this is not the end of the story.

### 4.2 The input impedance of a sEMG amplifier: its effect on the power line interference.

Unfortunately, undesired small capacitors are always present between the power line wiring and the people in a room because these are facing surfaces. They are often called "parasitic capacitors" or "parasitic capacitances" or "stray capacitors" and are of the order of 2-10 $\mathrm{pF}(1 \mathrm{pF}=10-12 \mathrm{~F})$. Other stray capacitors exists between a person and ground. These are depicted as $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$, respectively, in Fig. 4. Additional stray capacitors are related to the equipment (C3 and C 4 in Fig. 4). The impedances of these capacitors at the power line frequency form a capacitive voltage divider whose dividing factor results in a $\mathrm{Vcm}_{\mathrm{cm}}$ of a few Volt.
At the frequency of 50 Hz an input capacitor of $5-10 \mathrm{pF}$ presents an impedance of $637 \mathrm{M} \Omega-318$ $\mathrm{M} \Omega$ (see eq. 9 and Table 1) while the input resistor will present an impedance of $1 \mathrm{G} \Omega$ to $10 \mathrm{G} \Omega$ or even higher.


Fig. 4. Schematic representation of a sEMG amplifier connected to a subject. The stray capacitances ( $C_{1}, C_{2}, C_{3}$, and $C_{4}$ ) to ground and to the power line, the electro-skin impedances $\left(Z_{e l}, Z_{e 2}\right)$, and the amplifier input impedances $\left(Z_{i}\right)$ are shown. The common-mode voltge $v_{c m}$ is applied to two voltage dividers, one formed by $Z_{e 1}$ and $Z_{i}$ and the other by $Z_{e 2}$ and $Z_{i}$. Since $\Delta Z_{e}=$ $\left(Z_{e l}-Z_{e 2}\right)$ is, in general not zero, a vaiff will be generated and amplified just as the sEMG signal (eq. 14 and 15).

Eq. 12 can now be written as

$$
\begin{equation*}
V_{\text {out }}=A_{c m} V_{c m}+A_{d} V_{c m}\left(Z_{\mathrm{e} 1}-Z_{\mathrm{e} 2}\right) / Z_{i}+A_{d} \text { VEMG } \tag{14}
\end{equation*}
$$

By dividing both sides of eq. 14 by $\mathrm{A}_{\mathrm{d}}$ we have the expression of the input "equivalent" voltage that accounts for the input undesired interference component and the input EMG component:

$$
\begin{equation*}
V_{\text {in-eq }}=V_{\text {out }} / A_{d}=V_{c m}\left(A_{c m} / A_{d}+\Delta Z_{\mathrm{e}} / Z_{i}\right)+V_{\mathrm{EMG}} \tag{15}
\end{equation*}
$$

where $\Delta \mathrm{Z}_{\mathrm{e}}=\left(\mathrm{Z}_{\mathrm{e} 1}-\mathrm{Z}_{\mathrm{e} 2}\right)$. Since the frequency of vcm is 50 Hz the undesired interference component in eq. 14, $\mathrm{V}_{\mathrm{cm}}\left(\mathrm{Acm}_{\mathrm{cm}} / \mathrm{Ad}_{\mathrm{d}}+\Delta \mathrm{Ze} / \mathrm{Zi}_{\mathrm{i}}\right.$ ), must be minimized for $\mathrm{f}=50 \mathrm{~Hz}$ (or 60 Hz ). This can be done by:

1. Reducing Vcm , that is by minimizing the stray coupling with power cables and wirings.
2. Reducing $\mathrm{Acm} / \mathrm{Ad}$, that is by selecting an amplifier with a CMRR of at least $80-90 \mathrm{~dB}$.
3. Reducing $\Delta \mathrm{Z}_{\mathrm{e}}$, that is by proper skin treatment that would lower $\mathrm{Z}_{\mathrm{e} 2}$ and $\mathrm{Z}_{\mathrm{e} 1}$ and make them similar.
4. By choosing an amplifier with high $\mathrm{Z}_{\mathrm{i}}$ at 50 Hz , which implies a low $\mathrm{C}_{\mathrm{i}}$ (Fig 3c), possibly $\mathrm{C}_{\mathrm{i}} \leq 10 \mathrm{pF}$ resulting in $\mathrm{Zi}_{\mathrm{i}} \geq 320 \mathrm{M} \Omega$.

Point 4 of the above list is important and provides the reason why sEMG amplifiers should have a high $\mathrm{Z}_{\mathrm{i}}$ at 50 Hz and therefore a low Ci .

## 5. Conclusions

The main reason for choosing a sEMG amplifier with a high input impedance (at 50 Hz ) is to reduce the power line interference, not to reduce the attenuation of the sEMG signal which is usually very small and negligible, unless the electrode-skin impedance is very high (in the order of many $\mathrm{M} \Omega$ ).
The manufacturer should either provide the amplifier input impedance at 50 Hz or the value of the input resistor $\mathrm{R}_{\mathrm{i}}$ and of the input capacitor $\mathrm{C}_{\mathrm{i}}$. In this second case, the input impedance can be calculated using eq. 13 with $\mathrm{f}=50 \mathrm{~Hz}$.
Examples of incorrect indications:
Input resistance $=$ or $>10 \mathrm{G} \Omega$ (does not say anything about input impedance).
Input impedance $=$ or $>5 \mathrm{G} \Omega$ (not realistic, too high and without indication of the frequency of measurement. Very likely, this is the input resistance.)

Some sEMG (or ECG or EEG) instrumentation includes the option to measure the electrode-skin impedance $\mathrm{Ze}_{\mathrm{e}}$ but rarely indicate at which frequency such measurement is performed. Since $\mathrm{Z}_{\mathrm{e}}$ has capacitive components, its value depends on the frequency used for the measurement. Of course, a sinusoidal waveform must be used for this measurement.

## 5. Reading material

Some textbooks, reviews and tutorials dealing with the issue of input impedance and attenuation of power line interference are indicated below. See also teaching module 4 in this website at https://www.robertomerletti.it/en/emg/material/teaching/module6.

1. Merletti R., Parker P.A. (edts.), Electromyography: Physiology, engineering and non invasive applications, IEEE Press / J Wiley, USA, 2004. Chapter 5, pg . 107-132.
2. Merletti R, Aventaggiato M, Botter A, Holobar A, Marateb HR, Vieira TMM. Advances in surface EMG: recent progress in detection and processing techniques. Crit. Rev. Biomed. Eng. 2010; 38: 305-345.
3. Piervirgili G, PetraccaF,Merletti R. A new method to assess skin treatments for lowering the impedance and noise of individual gelled Ag -AgClelectrodes. Physiol. Measurement. 2014; 35: 2101-2118.
4. Merletti R, Farina D. (edts) Surface Electromyography: physiology, engineering and applications, IEEE Press / J Wiley, USA, 2016, Chapter 3, pg. 54-90.
5. Merletti R., Muceli S., Tutorial. Surface EMG detection in space and time: best practices. Journ. of Electromyography and Kinesiology, 2019; 49: doi.org/10.1016/j.jelekin.2019.102363
6. Merletti R., Cerone G.L., Tutorial. Surface EMG detection, conditioning and pre-processing; best practices. In press in J. of Electrom. and Kinesiol. 2020.
